# Quantitative Evaluation

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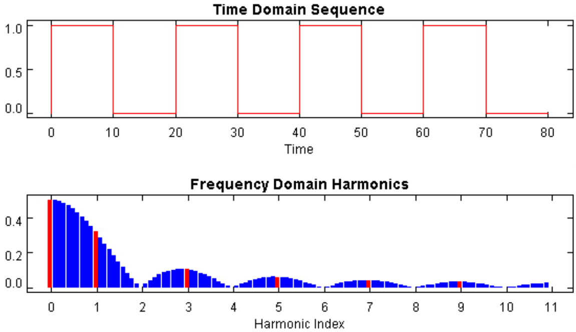
- We had lots of trouble with intermodulation distortion that affected our measurements
- Finally it seems that we have them under control – but we have to do as much checks as possible to test the reliability of our data
- The quantitative Evaluation provides another excellent reliability check for the microwave transmission method

- Determined graphically the phase modulation signals [dBc] for the 1$^{st}$, 2$^{nd}$ and 3$^{rd}$ injection
- These measured values are related to the integrated electron cloud density which means, they include the build up, the peak value and the decay of the electron cloud.
- We are only interested in the peak value $\Rightarrow$ corrections are needed to determine it

- We have to take into account the duty cycle of the machine (assumption: rectangular shaped signal)
- Test measurements were done to investigate the influence/effect of the duty cycle
- We used a signal generator delivering the phase modulation signal. This generator was triggered externally by another generator creating a rectangular signal of adjustable duty cycle. The triggered signal was displayed on a VSA in phase demodulation.
- As expected, we saw the maximum signal for a duty cycle of 50%
- Furthermore, we determined the loss for lower cycles

- A Fourier analysis for a rectangular signal was done using an online tool (http://www.eecircle.com/applets/001/001.html)
- We used the fundamental frequency (harmonic index n= 1) for the maximum duty cycle (50%) as reference:




- first harmonic: 0.318

- We determined the correction for other duty cycles (20, 30 and 40%) analytically by calculating the difference between the values of the fundamental frequency and the one of the reference value at 50% in dB
- A comparison with the test measuremet result showed a good agreement:

| Duty cycle [%] | Measured correction factor [dB] | Calculated correction factor [dB] |
|---|---|---|
| 40 | 0 | 0.47 |
| 30 | 1 | 1.8 |
| 20 | 4 | 4.6 |

- In case of LHC nominal beam in the SPS the batches are spaced with $\approx$ 225 ns and each batch consists of 72 bunches equally spaced with 25 ns
- Simulations of the electron cloud distribution for this beam type show, that the peak value of the electron cloud density lasts for 1 $\mu$s over the first batch and $\approx$ 1.6 $\mu$s for each following batch

- We calculated the duty cycle of the machine as follows
- The duration of the electron cloud signal of one turn was related to the machine circumference. For one batch this would be:

$$\text{Duty cycle} = \frac{1 \cdot 10^{-6}\,\text{s}}{6911\,\text{m}/3 \cdot 10^{8}\text{ms}^{-1}} = 4.34\,\%$$

- The results for two, three and four batches are:

| Number of batches | Duty cycle [%] |
|---|---|
| 1 batch | 4.34 |
| 2 batches | 11.3 |
| 3 batches | 18.23 |
| 4 batches | 25.2 |

- The correction factors for the above duty cycles were calculated using the online tool:

| Number of batches | Correction factor [dB] |
|---|---|
| 1 batch | 17.4 |
| 2 batches | 9.3 |
| 3 batches | 5.5 |
| 4 batches | 2.9 |

- Induced phase shift of a microwave going through plasma:

$$\Delta\varphi = \frac{L\,\omega_{\mathrm{p}}^2}{2\mathrm{c}\,(\omega^2 - \omega_{\mathrm{c}}^2)^{\frac{1}{2}}} \tag{1}$$

where $\omega$ is the injected frequency, L the transmission length, $\omega_{\mathrm{c}}$ the cutoff frequency of the waveguide, c the speed of light and $\omega_{\mathrm{p}}$ the plasma frequency

- $\Delta\varphi$ is related to the electron cloud density via the plasma frequency:

$$\omega_{\mathrm{p}} = \sqrt{\frac{n_{\mathrm{e}}\mathrm{e}^2}{\epsilon_0 \mathrm{m_e}}} \cong 56.4\sqrt{n_{\mathrm{e}}} \tag{2}$$

where e is the electron charge, $\epsilon_0$ the permittivity in free space and $\mathrm{m_e}$ the electron mass

- Using the previous formulas with the corrected measurement values, one obtains a value of $1.5 \cdot 10^{12}\,\mathrm{m}^{-3}$ for the electron cloud peak density
- This is a reasonable result

- The value obtained with the microwave transmission method for the peak electron cloud density is reasonable which is a further criterion for us to remain confident in the data delivered by this method.